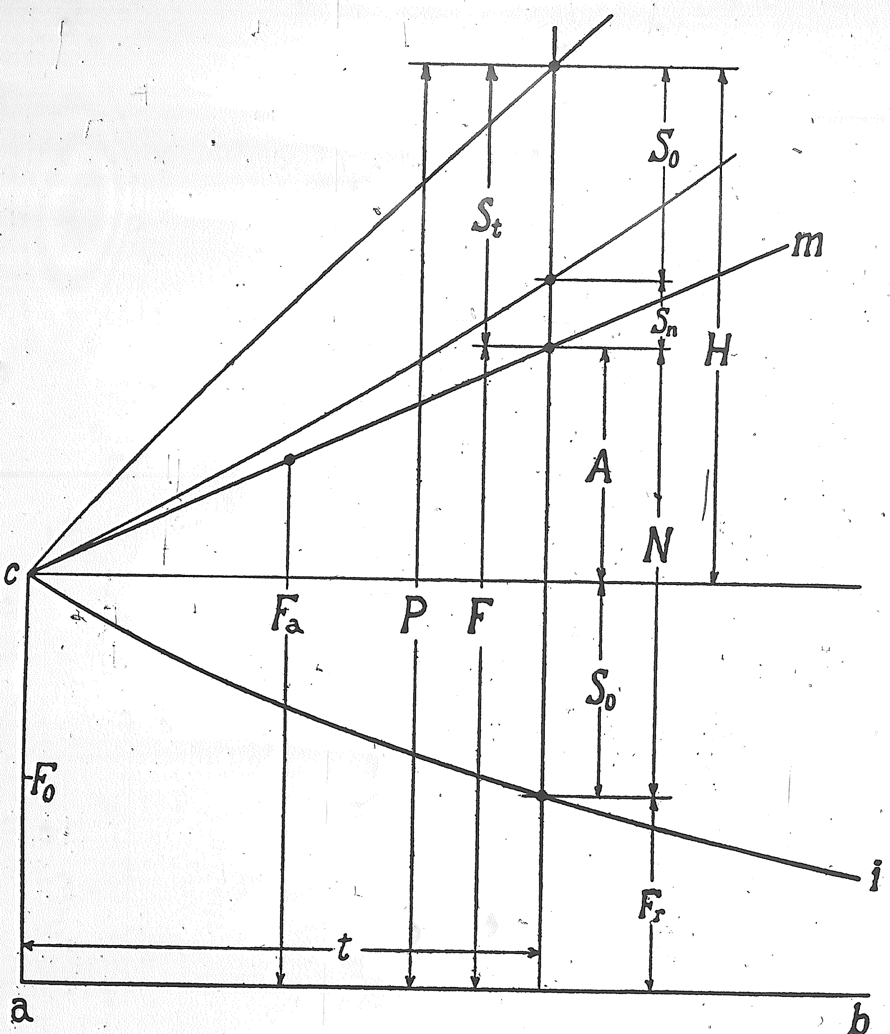

FIG. 4

In Fig. 4 all the elements that enter into our analysis have been represented. Thus, besides the elements already referred to, we have

$S_o = F_o - F_r$ = number of separations from the original force, and by substituting the value of F_r in formula (1), this becomes

$$S_o = F_o\left(1 - \frac{1}{\varepsilon^{rt}}\right). \qquad (4)$$

$N = A + S_o$ = total number of new workers on the increased force F, and by substituting the value of A in formula (2) and the value of S_o in formula (4), this becomes

$$N = F_o\left(i\,t + 1 - \frac{1}{\varepsilon^{rt}}\right). \qquad (5)$$

F_a = average force during the time period t.

For the simple straight line law of increase assumed for the force, we have

$F_a = \frac{1}{2}(F_o + F)$, which, when we substitute the value of F in formula (3), becomes

$$F_a = \tfrac{1}{2}\,[F_o + F_o(1 + i\,t)] = F_o\left(1 + \frac{i\,t}{2}\right) \qquad (6)$$

If by n we now designate the rate at which separations take place from among the new workers N on the force, the total number of such separations during the time period t can be shown to be[1]

$$S_n = F_a\,n\,t - \frac{n}{r}S_o \qquad (7)$$

Adding formulas (4) and (7) we finally get the total number of separations that have taken place, to be

$$S_t = S_o + S_n = S_o + F_a\,n\,t - \frac{n}{r}S_o = F_a\,n\,t - \frac{n-r}{r}S_o \qquad (8)$$

Further

$H = A + S_t$ = total new hires during the period, and

$P = F_o + H$ = total number of names on the payroll during the period.

If we now calculate the total labor turnover for the period of time t by the universally accepted method for an increasing force, viz., making the numerator of the labor turnover fraction equal to the total separations for the period (which for both an increasing and a constant force equals replacements), and using the average force as given by formula (6) as denominator, we get

Total labor turnover

$$L_t = \frac{S_t}{F_a} = \frac{F_a\,nt - \frac{n-r}{r}S_o}{F_a} = n\,t - \frac{n-r}{r}\cdot\frac{S_o}{F_a}, \qquad (9)$$

which divided by t finally gives the labor turnover for the unit time period (rate of labor turnover for the period t).

$$L_1 = n - \frac{n-r}{rt}\cdot\frac{S_o}{F_a} \qquad (10)$$

In this formula all evidence of its having been derived by considering an increasing force only, has entirely disappeared. To be sure, for an increasing force the average force F_a during the time period t would be greater than the force at the beginning of the period (original force), and for a decreasing force it would be smaller than F_o; but in nominally applying the formula we need to know only the magnitude of the average force F_a.

Hence, as I see it, this formula compels us, for the sake alone of the respect we owe mathematical logic and consistency, to figure labor turnover by considering separations and not replacements, in the case of a decreasing force also.

If in formula (10) we make no attempt at distinguishing between the average rate n at which new workers leave, and the average rate r at which workers of the original force leave, it reduces itself to

$$L_1 = n = r \qquad (11)$$

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[1]Formula (7) may be derived as follows: Let δS_n = number of separations from the new workers N which takes place at the rate n during the time element δt following the time period t, then

$$\delta S_n = Nn\delta t = F_o\left(i\,t + 1 - \frac{1}{\varepsilon^{rt}}\right)n\delta t$$
$$= F_o\,[n\,i\,t\,\delta t + n\,\delta t - n\varepsilon^{-rt}\,\delta t]$$
$$= F_o\left[n\,i\,t\,\delta t + n\delta t - \frac{n}{r}\varepsilon^{-rt}\,\delta(-rt)\right].$$

Integrating we then get, as $\int \varepsilon^x\,\delta_x = \varepsilon^x + C$,

$$S_n = F_o\left[ni\int t\,\delta t + n\int \delta t + \frac{n}{r}\int \varepsilon^{-rt}\,\delta(-rt)\right]_{t=o}^{t=t}$$
$$= F_o\left[ni\frac{t^2}{2} + n\,t + \frac{n}{r}\varepsilon^{-rt}\right]_{t=o}^{t=t}$$
$$= F_o\left[\left(1 + \frac{i\,t}{2}\right)n\,t - \frac{n}{r}\left(1 - \frac{1}{\varepsilon^{rt}}\right)\right]$$

By formulas (4) and (6) this reduces to

$$S_n = F_a\,n\,t - \frac{n}{r}S_o. \qquad (7)$$