the mathematical expressions for the straight line *ch* and the curve *ci* will both, for a short period, represent quite closely the approximate law of depletion of the force, as this has been illustrated by the curve *cd*.

Further, if we knew the extent to which an original force would deplete itself through a somewhat longer period, but still one far short of the entire depletion period, a curve of the nature of *ci* in Fig. 1 might be drawn to coincide with the true curve of depletion at the point representing such period of partial depletion, when it then, most undoubtedly, would also be found to coincide very closely with the true curve of depletion between this point and the starting point, as illustrated by the diagram in Fig. 2.

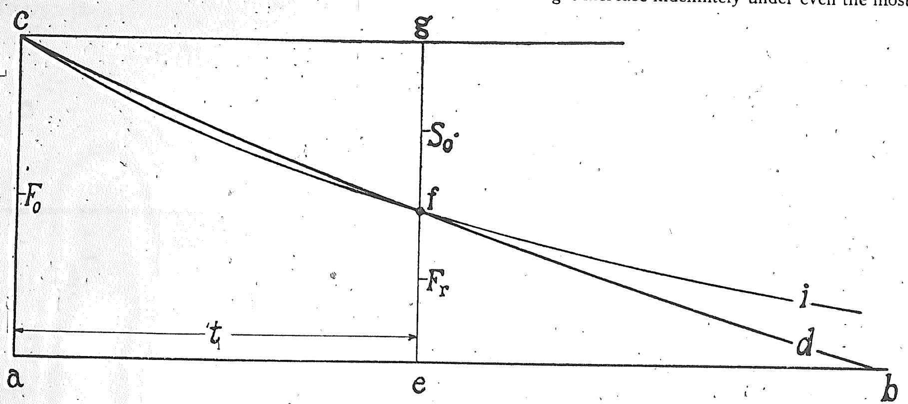

Fig. 2

Hence, for the purpose of this discussion, it will be assumed that the curve representing the gradual depletion of an original force may be approximated by a curve representing a law of depletion similar to that on which Table 1 was constructed. This curve is a logarithmic curve[1] whose equation is

$$F_r = \frac{F_o}{\varepsilon^{rt}} \qquad (1)$$

ε being the base of the Naperian system of logarithms, r the rate of depletion (separation) and t the period of time during which the original force F_o is reduced or depleted to F_r.

We must next determine upon some law in accordance with which we will wish to increase the original force by hiring new workers; first, to replace the separations from this original force as fast as they occur; and secondly, to increase the thus replenished original force to the point desired at any one time, in spite of the separations that also take place from among the newly hired workers. Of course, a law to govern the increase of a working force must either be in accordance with some decreasing rate of increase, or else in accordance with some constant or even increasing rate of increase, which must then, sooner or later, come to a more or less sudden stop; for it is inconceivable that a force might increase indefinitely under even the most favorable conditions of industrial expansion.

[1]Formula (1) may be derived as follows: The original force F_o having been reduced to F_r at the end of the time period t, and the decrease taking place at the rate r as figured on the magnitude of F_r at any time, the decrease (negative increase) during the time element δt following the end of the time period t, may be written

$$\delta F_r = F_r(-r)\delta t, \text{ and } \frac{\delta F_r}{F_r} = -r\,\delta t$$

Integrating, we then get, as $\int \frac{\delta x}{x} = \log_e x + C$,

$$\log_e F_r = -rt + C.$$

To determine the constant of integration C we have $F_r = F_o$ for $t = o$, which makes $C = \log_e F_o$, and

$$\log_e F_r = -rt + \log_e F_o; \text{ and further}$$

$$\log_e F_o - \log_e F_r = \log_e \frac{F_o}{F_r} = rt,$$

which again gives $\varepsilon^{rt} = \frac{F_o}{F_r}$; and finally

$$F_r = \frac{F_o}{\varepsilon^{rt}} \qquad (1)$$

In view of the circumstance that labor turnover, while usually converted into the equivalent of an annual rate, is always calculated for shorter periods only, almost any general law of force increase that we may assume and which will readily lend itself to the mathematical treatment contemplated, will answer, just the same as we have already concluded that the law assumed for the separations from an original force must be near enough correct for use in our analysis.

In the diagram Fig. 3, let the curve *ck* represent an ideal law of increase of the original force $ac = F_o$ to the final desired force during the period of time $ab = T$. This curve is then, to begin with, quite closely approximated by its tangent *cm* at the point *C*, and represents the general mathematical law of increase of the working force that I have assumed for the present purpose.

If i designates the rate of increase of the force F as figured on the original force F_o, the increase of the force at the end of the time period t will be

$$A = F_o i t \qquad (2)$$

and the increased force will be

$$F = F_o + A = F_o(1 + it) \qquad (3)$$

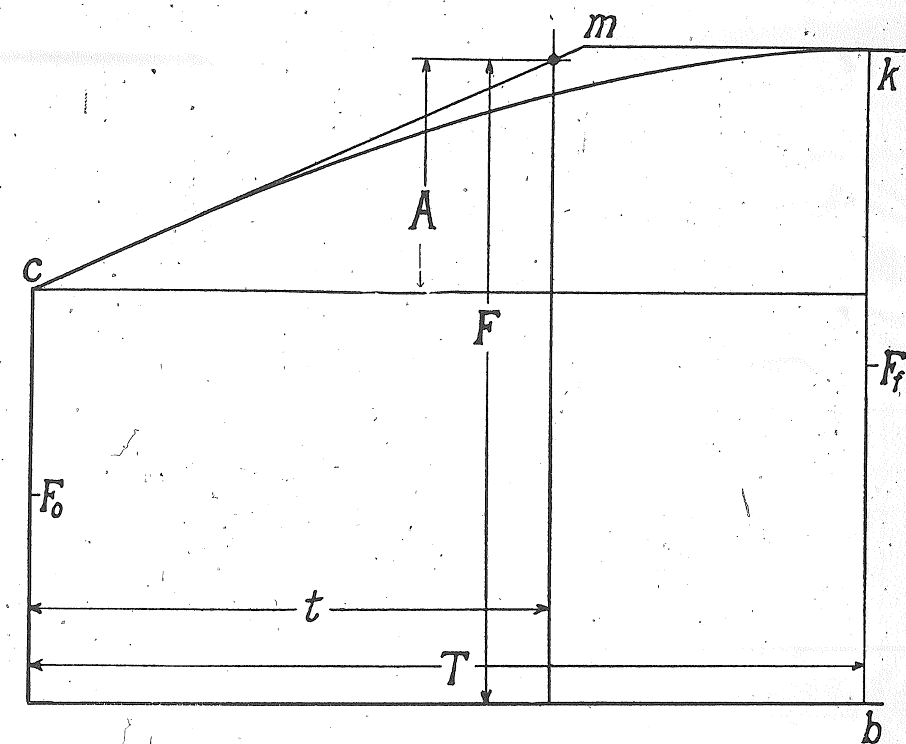

Fig. 3