are four principal elements which appear in practically all data with which analysis has to deal:

A. Cyclical movements.
B. Long time trend.
C. Seasonal variations.
D. Accidental movements.

The cyclical movements are those regular increases and decreases in general business which constitute the business cycle. These movements, which hardly ever occur at equal intervals, are of principal interest to the economist or student of business conditions, but every industry is concerned with the statistical methods which these specialists are developing to isolate and to examine these phases more intelligently.

The long time trend is an increase over a continuous number of years caused by development in magnitude of business, growth in population, etc., or it may be a decrease over a continuous period for similar reasons.

Within the year the intensity of business activities varies with the seasons, because of crop movements, conventional practices of trade, weather changes, and many other influences which relate to the particular season of the year.

The accidental movements include the various casual events which may affect an individual industry, or sometimes the entire business structure. Strikes, wars, new inventions are some of the unforeseen events.

Owing to the fact that conditions such as price changes, war periods, etc., are removed by various methods particularly adapted to each individual case, I shall dwell only on the elimination of long time trend and seasonal variations, reducing the figures to cyclical and accidental movements.

The long time trend may be determined by fitting a straight line to the annual averages of the monthly items under analysis; in other words, fitting the most probable straight line to the data. From this line can be calculated the average monthly increment. The method used in our study to determine the long time trend is known as the method of least squares, or it may be stated as follows:—the most probable value of a measured quantity is that value which makes the sum of the squares of all the differences between the various observations and the most probable value, a minimum.

In order to obtain the most probable straight line to fit to a set of points, it is necessary to measure some function or functions of a straight line, from which the line desired can be derived. Since a straight line may be represented by an equation "$y = mx + b$," if the most probable value of "m" and "b" is found from given coordinates "x's" and "y's" of the various points, it is but a simple matter to draw the most probable line from these points. The method of deriving the line is based on the principle, as stated before, that the sum of the squares of the differences is a minimum.

There are two methods of arriving at the line of long time trend or normal growth:

1. A long formula (the Harvard method) to find the value of "m," the slope of the line of long time trend, and "b," its "y-intercept," as follows:

$$m = \frac{n\Sigma xy - \Sigma x\Sigma y}{n\Sigma x^2 - \Sigma x\Sigma x} \qquad b = \frac{\Sigma x\Sigma x^2 - \Sigma x\Sigma xy}{n\Sigma x^2 - \Sigma x\Sigma x}$$

2. A shorter formula (the American Telephone and Telegraph Company method) derived from the long formula as follows:

$\Sigma x = 0$. (See Table 1.) Then

$$m = \frac{\cancel{n}\Sigma xy - \overset{0}{\cancel{\Sigma x}}\Sigma y}{\cancel{n}\Sigma x^2 - \underset{0}{\cancel{\Sigma x}}\,\underset{0}{\cancel{\Sigma x}}} = \frac{\Sigma xy}{\Sigma x^2} \text{ or Annual Increment.}$$

$$b = \frac{\overset{0}{\cancel{\Sigma x}}\Sigma x^2 - \overset{0}{\cancel{\Sigma x}}\Sigma xy}{n\Sigma x^2 - \underset{0}{\cancel{\Sigma x}}\,\underset{0}{\cancel{\Sigma x}}} = \frac{0}{n\Sigma x^2} = 0$$

∴ The short method

$$\text{Midpoint} = \frac{\Sigma y}{n}$$

$$\text{Annual Increment} = \frac{\Sigma xy}{\Sigma x^2} \quad \text{Odd number of years.}$$

$$= \left.\frac{\Sigma xy}{\frac{\Sigma x^2}{2}}\right\} \text{Even number of years.}$$

$$\text{Monthly Increment} = \frac{\frac{\Sigma xy}{\Sigma x^2}}{12} \text{ or } \frac{\text{A. I.}}{12}$$

I shall now endeavor to explain in detail how we use this short method to determine the long time trend of our business. The first problem is to select a period of years which will characterize the growth of the company. The period considered should be no shorter than a business cycle, so that unusual influences of cyclical changes may be equally weighted, and should begin and end with the same oscillation of the business cycle.

After selecting a period, the first move is to list the annual averages of monthly items by years in a column "Y," as shown in Table 1. The midyear of the period of time in the study is found and the distance from the midpoint of each year from this point is placed in column "X." If the number of years in the study is odd, these distances of the midpoint of each year from the midyear are measured

Table No. 1

Alpha Products—Odd Number of Years

Year	*Dozens (y)*	*x*	*x²*	*xy*	*Trend*
1912	27,350	–6	36	–164,100	24,361.7
1913	26,150	–5	25	–130,750	24,819.4
1914	21,600	–4	16	– 86,400	25,277.1
1915	24,000	–3	9	– 72,000	25,734.8
1916	26,850	–2	4	– 53,700	26,192.5
1917	25,600	–1	1	– 25,600	26,650.2
1918	26,350	0	0	0	27,107.9
1919	32,950	1	1	32,950	27,565.6
1920	27,400	2	4	54,800	28,023.3
1921	19,100	3	9	57,300	28,481.0
1922	32,800	4	16	131,200	28,938.7
1923	35,400	5	25	177,000	29,396.4
1924	27,100	6	36	162,600	29,854.1
TOTAL	352,650	0	182	83,300	

$$\text{Midpoint} = \frac{\Sigma y}{n} = \frac{352,650}{13} = 27,127$$

$$\text{Annual Increment} = \frac{\Sigma xy}{\Sigma x^2} = \frac{83,300}{182} = 457.7$$

$$\text{Monthly Increment} = \frac{\text{A. I.}}{12} = \frac{457.7}{12} = 38.1$$

in units of one year. On the other hand, if the number of years is even, the distances are measured in units of half years (see Table 2). Likewise, all those years preceding the midyear in column "X" are given minus signs and those following, plus signs. The square of these "X" items is recorded in column "X²." The midpoint of the line of long time trend in column "Y," is found by dividing the sum of average monthly items in column "Y" by the number of years under analysis. The annual increment, that is, the amount by which the line of long time trend increases from year to year, is the quotient of the sum of the "XY" items, taking into consideration the plus and minus signs, divided by the sum of "X²" items when the years are odd. When the years are even, the annual increment is found by dividing the sum of "XY" items by ½ of the sum of "X²" items, as previously explained. The monthly increment is 1/12 of the annual increment.

The next step is to find the monthly periods for the ordinates of the long time trend. For an odd number of years, the midpoint will fall between June and July of the middle year. Conversely, for an even number of years the midpoint will fall between December and January of the middle years. By subtracting ½ of the monthly increment from the midyear, the trend ordinate for June of the middle year is found. To find June trend ordinate for years preceding the midyear, subtract successively the an-

Table No. 2

Alpha Products—Even Number of Years

Year	*Dozens (y)*	*x*	*x²*	*xy*	*Trend*
1912	27,350	–13	169	–355,550	24,593.9
1913	26,150	–11	121	–287,650	25,047.1
1914	21,600	– 9	81	–194,400	25,500.3
1915	24,000	– 7	49	–168,000	25,953.5
1916	26,850	– 5	25	–134,250	26,406.7
1917	25,600	– 3	9	– 76,800	26,859.9
1918	26,350	– 1	1	– 26,350	27,313.1
1919	32,950	1	1	32,950	27,766.3
1920	27,400	3	9	82,200	28,219.5
1921	19,100	5	25	95,500	28,672.7
1922	32,800	7	49	229,600	29,125.9
1923	35,400	9	81	318,600	29,579.1
1924	27,100	11	121	298,100	30,032.3
1925	30,000	13	169	339,000	30,485.5
TOTAL	382,650	0	910	203,950	

$$\text{Midpoint} = \frac{\Sigma y}{n} = \frac{382,650}{14} = 27,332$$

$$\text{Annual Increment} = \frac{\Sigma xy}{\frac{(\Sigma x^2)}{(2)}} = \frac{203,950}{\frac{910}{2}} = 453.2$$

$$\text{Monthly Increment} = \frac{\text{A. I.}}{12} = \frac{453.2}{12} = 37.8$$

nual increment from June ordinate of the middle year, then add the annual increment successively to get the June trend for following years. Having found the June ordinate for each year, fill in the ordinates for the remaining months by adding the monthly increment successively to the already found ordinates. Figure 2 shows actual monthly plottings of Alpha Products revealing the line of long time trend.

There are two ways of arriving at seasonal variations. One method is called Macaulay's or Federal Reserve and another is Persons' Link Relative method. The former method operates best with figures which are a widely fluctuating series over a period of years. The latter method, which is equally good, seems to depend for accuracy on either more moderate fluctuations or a much longer period of years. In an industry similar to ours, it is felt that the fluctuations vary to a great extent during the course of the year. We have used Macaulay's method and it has proved to be the best for us.