LABOR TURNOVER[1]

A MATHEMATICAL DISCUSSION

By
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BELIEVING that some kind of mathematical analysis might be made to throw light on the main point in the controversy about how labor turnover ought to be calculated, I recently set to work on such analysis as the mathematics within my reach would enable me to make; and having obtained some interesting results, I now wish to present them to the Taylor Society.

So far as I know labor turnover is by everybody presented as a percentage arrived at by dividing the denominator of a fraction into the numerator of that fraction, and the controversy centers on what elements should enter into the two terms of this fraction.

Thus, one set of advocates seems to contend that the average working force for the period considered, as ascertained from the number of workers on the total payroll, should be made the denominator of the labor turnover fraction, while another set contends that the working force so to be used, should include only the average number of workers actually in attendance during that period.

As to the numerator, some advocates contend that this should consist of the total number of separations during the period considered, while others contend that only the number of actual replacements of the separated workers should be used. This makes possible altogether four distinct ways of calculating labor turnover, all of them probably in use; and it is needless to say that until only one of these is adopted as a standard, general comparisons cannot be made of labor turnover in different plants or industries. Of course, as is no doubt universally recognized, when the average working force either remains constant or increases during several periods, these alternative numerators are alike, for then all separations are replaced.

My mathematical analysis is entirely confined to throw light on the question of which of the two alternative numerators should be used, and the conclusion reached is in a broad way in favor of the total separations as against the replacements only, regardless of whether the total force is increasing or decreasing. Incidentally it has also led me to believe that, as a matter of consistency, the average total payroll should be made the denominator as against the average total attendance only; for the workers that do not attend during the period considered are not separated so long as their names are retained on the payroll.

In this analysis I consider a working force that is increasing according to some simple mathematical law, through the hiring of more workers than the increase of the force directly demands, on account of the separations that constantly occur and which must first be replaced. These separations I have divided into two classes; viz., separations from the force as it was at the beginning of the period considered (the original force) and separations from among the new workers since hired. To be sure, the new workers soon become more or less amalgamated with the remaining workers of the original force so that no definitely determinable distinction can long be made between a new and an old worker; but by assuming such a division, and also assuming a different rate of separation for each, I figure that I am closer to what actually takes place than by assuming one rate of separation for the total working force at all intervals during the period considered; and when this period does not exceed that usually employed in periodic labor turnover calculations of the one kind or the other, the assumption is legitimate enough for a mere mathematical theory.

Mathematically considered there is no essential difference between an increasing and a decreasing force, a rate of decrease being simply a negative or minus rate of increase, and a constant force at the same time being one whose rate of increase is 0, or the dividing line between the positive and the negative rates of increase. For this reason, any truly mathematical expression for labor turnover that may be agreed upon as being correct for an increasing force, must necessarily also hold good if the rate of the increase

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gradually diminishes; first to 0, when the force then momentarily becomes stationary or constant; next, becomes negative, or, what is then the same thing, becomes a positive rate of decrease, and, with it, the force becomes a decreasing one.

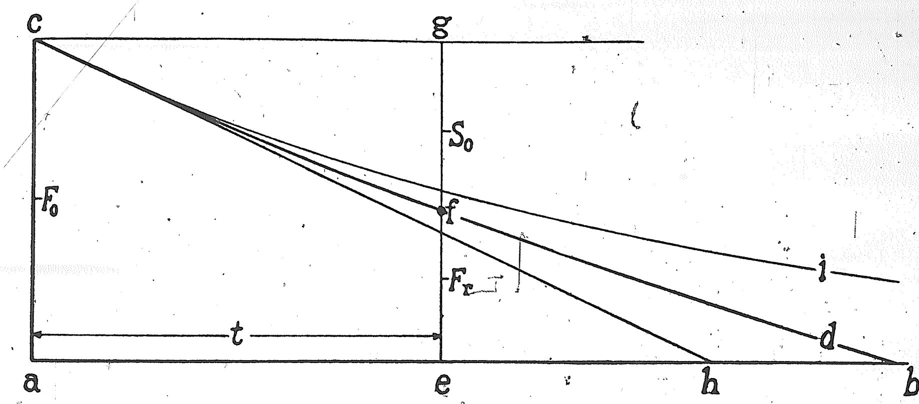

Fig. 1

In the diagram Fig. 1, the vertical line $ac=F_o$ represents the working force at the beginning of the period to be considered (the original force), and the vertical distance $ef=F_r$ represents similarly the remaining portion of the original force at the end of the period of time t (days, weeks or months) which is represented by the horizontal distance ae along the base line ab, while the vertical distance $fg=S_o$ then represents the separations from the original force during the same time t. The first question is then: What mathematical law can we assume to express near enough correctly the relation between the time t and the decreased force F_r in view of the fact that no statistics have probably ever been compiled to show, for even a single plant, how an original force gradually decreases to O through a term of years depending on the degree of "mutual employment satisfaction" and on such unpreventable causes as death, protracted illness, etc.?

Let us assume, for example, an original force F_o of 1000 workers, and that, to begin with, these separate themselves at the rate of 100 each month. Then, at the end of the first month, there would be left a force of 900 workers. If this rate of 100 per month should keep up until the force is entirely depleted, this would evidently take place in just $\frac{1000}{100} = 10$ months. This, however, would be an absurd law of depletion of such a force, for it would mean that all of the 100 workers remaining at the end of the 9th month would be separated during the 10th month.

Let us then assume, on the other hand, that the rate of 100 leaving the first month out of an original total of 1000, keeps up indefinitely in the same proportion only, then we would have:

TABLE 1.

At beginning of	Size of Force	Separations during
1st month	1000	100
2nd month	900	90
3rd month	810	81
4th month	729	72.9

and a complete depletion of the original force would never take place, which again is an impossibility, for ultimately the workers must all separate at least through death.

These two extreme, and hence absurd, assumptions are respectively represented in the diagram Fig. 1, by the straight line ch and the curve ci, both of which, to begin with, coincide quite closely with the curve cd which is located between them in a manner to represent more nearly the correct law of depletion of the force.

It will now be realized that, as a mathematical law of depletion can only be an approximate one (just as is the mortality law used by life insurance companies),